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SPECTRAL DECOMPOSITION OF GREEN'S TENSOR OF THE DYNAMIC PROBLEM OF THE THEORY OF ELASTICITY IN CYLINDRICAL COORDINATES*

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Formulas are obtained for Green's function of the dynamic problem of the theory of elasticity in an unbounded isotropic medium describing, in cylindrical coordinates R, φ and z, the Fourier transforms of this function in t, and in coordinates φ and z. The dependence on R is retained in explicit form and this makes it possible to use the results obtained to solve boundary-value problems in circular cylindrical coordinates. A special case of the spectral components of the field of displacement rates of the points of the medium and of the stress field of the system of dislocation loops moving in arbitrary manner, are discussed.

The equation of motion of an elastic medium can be written in curvilinear coordinates in covariant form /1/

$$(\rho g^{\alpha\delta}\partial^2/\partial t^2 - \lambda^{\alpha\beta\gamma\delta}\nabla_\beta\nabla_\gamma)\, u_\delta = f^\alpha \tag{1}$$

where $u_\alpha(\mathbf{y}, t)$ is the displacement field of the points of the medium in the system of generalized coordinates $\{y^\alpha\}$ at the instant of time t; $f^\alpha(\mathbf{y}, t)$ is the volume force density, ρ is the density of the medium, $g^{\alpha\beta}$ is the metric covariant tensor /2/, ∇_α is the covariant tensor derivative /1, 2/ and $\lambda^{\alpha\beta\gamma\delta}$ is the quadruply contravariant tensor of the elastic moduli of the medium /1, 3/. In the case of an isotropic medium, to which we shall confine the present discussion, to this tensor will have the form /1/

$$\lambda^{\alpha\beta\gamma\delta} = \rho(c_l^2 - 2c_t^2)\, g^{\alpha\beta}g^{\gamma\delta} + \rho c_t^2 (g^{\alpha\gamma}g^{\beta\delta} + g^{\alpha\delta}g^{\beta\gamma}) \tag{2}$$

(c_l and c_t denote the longitudinal and transverse speeds of sound).

Let us introduce the twice-covariant Green's tensor $G_{\alpha\beta}^{(0)}(\mathbf{y}, \mathbf{y}'; t - t')$ satisfying the equation

$$(\rho g^{\alpha\delta}\partial^2/\partial t^2 - \lambda^{\alpha\beta\gamma\delta}\nabla_\beta\nabla_\gamma)\, G_{\delta\mu}^{(0)} = \Omega_0^{-1}\delta_\mu^\alpha\delta(\mathbf{y} - \mathbf{y}')\,\delta(t - t') \tag{3}$$

where $\Omega_0 = h_1h_2h_3$ (h_α are the Lamé parameters), and δ_β^α is the Kronecker delta. After this we can write the solution of Eq.(1) in the form /4, 5/

$$u_\alpha(\mathbf{y}, t) = \int_{-\infty}^{t} dt' \int d\Omega' \Gamma_{\alpha\beta}(\mathbf{y}, \mathbf{y}'; t - t')\, f^\beta(\mathbf{y}', t') \tag{4}$$

Here $d\Omega$ is the volume element in y-space, and

$$\Gamma_{\alpha\beta}(\mathbf{y}, \mathbf{y}'; t - t') = G_{\alpha\mu}^{(0)}(\mathbf{y}, \mathbf{y}'; t - t')\, P_\beta^\mu(\mathbf{y} \mid \mathbf{y}') \tag{5}$$

is Green's function of the dynamic problem of the theory of elasticity in an unbounded medium in coordinates $\{y^\alpha\}$, where $P_\beta{}^\mu(\mathbf{y} \mid \mathbf{y}')$ is the projection operator mapping the components of the vector defined in the basis $\mathbf{e}_\mu(\mathbf{y})$ at the point $\mathbf{y}$, into the basis $\mathbf{e}_\beta(\mathbf{y}')$ at the point $\mathbf{y}'$. For a Cartesian system of coordinates $P_\beta{}^\gamma = \delta_\beta{}^\gamma$ and in curvilinear coordinates,

$$P_{\alpha\beta}(\mathbf{y} \mid \mathbf{y}') = \frac{\partial \mathbf{r}}{\partial y^\alpha}\frac{\partial \mathbf{r}'}{\partial y'^\beta} \tag{6}$$

where $\mathbf{r}$ and $\mathbf{r}'$ are the Cartesian radius vectors of the points at which the corresponding bases are constructed /1/. From (6) we obtain the obvious relation $P_{\alpha\beta}(\mathbf{y} \mid \mathbf{y}) = g_{\alpha\beta}(\mathbf{y})$.

In view of the representation (5), we must provide the following explanation. Green's function expresses the law of congruence between the vector quantities referred to the points of the space with different bases. Therefore, if we use, for example, the function $G_{\alpha\beta}^{(0)}$, whose components are defined in the basis $\mathbf{e}_\gamma(\mathbf{y})$, then, before calculating its convolution with the force $f^\beta(\mathbf{y}')$ which has components in the basis $\mathbf{e}_\gamma(\mathbf{y}')$, the latter must be projected

*Prikl.Matem.Mekhan., 55,3,428-432,1991

onto the basis $\mathbf{e}_\gamma(\mathbf{y})$ (otherwise the contraction operation will lose its meaning). Thus we arrive, once again, at relations (4) and (5), and $\Gamma_{\alpha\beta}$ can conveniently be regarded as Green's function, since unlike $G^{(0)}_{\alpha\beta}$, it satisfies the following reciprocity relations /4, 5/ in explicit form:

$$\Gamma_{\alpha\beta}(\mathbf{y},\mathbf{y}';t-t')=\Gamma_{\beta\alpha}(\mathbf{y}',\mathbf{y};t'-t). \tag{7}$$

Here the function $\Gamma_{\alpha\beta}$ satisfies, naturally, relations (3). The latter can be easily confirmed by taking into account the fact that the projector $P_{\alpha\beta}$ behaves, just as the metric tensor $g_{\alpha\beta}$, under differentiation, as a constant

$$\nabla_\gamma P_{\alpha\beta}=\partial P_{\alpha\beta}/\partial y^\gamma-P_{\mu\beta}\Gamma^\mu_{\alpha\gamma}=0$$
$$\nabla_\gamma' P_{\alpha\beta}=\partial P_{\alpha\beta}/\partial y'^\gamma-P_{\alpha\nu}\Gamma^\nu_{\beta\gamma}=0$$

where $\Gamma^\nu_{\alpha\beta}$ are Christoffel symbols /1, 2/ and the symbols ∇_γ and ∇_γ' denote differentiation with respect to y and y'. We note that the projector $P_{\alpha\beta}$ differentiates not as a covariant tensor of second rank, but as a set of covariant (basis) vectors, and this is obvious from its definition (6).

We know /6, 7/ that the function $G^{(0)}_{\alpha\beta}$ can be written as

$$G^{(0)}_{\alpha\beta}(\mathbf{y},\mathbf{y}';t-t')=L_{\alpha\beta}U(|\mathbf{y}-\mathbf{y}'|,t-t') \tag{8}$$

$$L_{\alpha\beta}=-\rho\{(c_l^2-c_t^2)\nabla_\alpha\nabla_\beta+\Box_l g_{\alpha\beta}\} \tag{9}$$

$$\Box_\lambda=\partial^2/\partial t^2-c_\lambda^2\nabla_\mu\nabla^\mu;\quad \lambda=l,t$$

The potential U satisfies the equation /7/

$$\rho^2\Box_l\Box_t U(|\mathbf{y}-\mathbf{y}'|;t-t')=\Omega_0^{-1}\delta(\mathbf{y}-\mathbf{y}')\,\delta(t-t') \tag{10}$$

The solution of Eq.(10) has the form /8/

$$U(y,t)=A\sum_{\lambda=l,t}^{(-)}\left(\frac{1}{c_\lambda}-\frac{t}{y}\right)\Theta\left(t-\frac{y}{c_\lambda}\right) \tag{11}$$

where $A=[4\pi\rho^2(c_l^2-c_t^2)]^{-1}$, $\Theta(t)$ is the Heaviside step function and the minus sign on the summation symbol means that we take the difference between the terms with $\lambda=l$ and $\lambda=t$. It is clear that the spectral expansion, which is of interest, can be conveniently carried out starting from relation (8), since the scalar potential (11) is invariant under any orthogonal coordinate transformation, and the transformation of the differential operator (9) in each specific case presents no fundamental difficulties.

All this makes the treatment of the problem in its general form more difficult. In what follows, we shall transfer to the special case of a cylindrical system of coordinates R, φ and z. Here we have $|y-y'|=[R^2+R'^2-2RR'\cos(\varphi-\varphi')]^{1/2}$, and the physical components of the projection operator are:

$$\bar P_{\alpha\beta}(\varphi-\varphi')=\left\|\begin{matrix}\cos(\varphi-\varphi') & \sin(\varphi-\varphi') & 0\\ -\sin(\varphi-\varphi') & \cos(\varphi-\varphi') & 0\\ 0 & 0 & 1\end{matrix}\right\| \tag{12}$$

(here and henceforth a prime on a vector and tensor denotes the physical component of the object in question). The physical components of the differential operator $L_{\alpha\beta}$ have the form

$$\bar L_{\alpha\beta}=\rho\delta_{\alpha\beta}\left[c_l^2\left(\frac{\partial^2}{\partial R^2}+\frac{1}{R}\frac{\partial}{\partial R}+\frac{1}{R^2}\frac{\partial^2}{\partial\varphi^2}\right)-\frac{\partial^2}{\partial t^2}\right]-$$
$$\frac{4\pi\rho}{A}\left\|\begin{matrix}\dfrac{\partial^2}{\partial R^2} & \dfrac{1}{R}\left(\dfrac{\partial^2}{\partial R\,\partial\varphi}-\dfrac{1}{R}\dfrac{\partial}{\partial\varphi}\right) & \dfrac{\partial^2}{\partial R\,\partial z}\\ \dfrac{1}{R}\left(\dfrac{\partial^2}{\partial R\,\partial\varphi}-\dfrac{1}{R}\dfrac{\partial}{\partial\varphi}\right) & \dfrac{1}{R}\left(\dfrac{1}{R}\dfrac{\partial^2}{\partial\varphi^2}+\dfrac{\partial}{\partial R}\right) & \dfrac{1}{R}\dfrac{\partial^2}{\partial\varphi\partial z}\\ \dfrac{\partial^2}{\partial R\partial z} & \dfrac{1}{R}\dfrac{\partial^2}{\partial\varphi\partial z} & \dfrac{\partial^2}{\partial z^2}\end{matrix}\right\| \tag{13}$$

The spectral expansion of the physical components of Green's function

$$\bar\Gamma_{\alpha\beta}(R,R',\varphi-\varphi',z;t)=(2\pi)^{-2}\sum_{m=-\infty}^{\infty}\int_{-\infty}^{\infty}d\omega\int_{-\infty}^{\infty}dk_z\bar\gamma^\omega_{\alpha\beta}(R,R'\,|\,m,k_z)\times$$
$$\exp[i(\omega t-k_z z)] \tag{14}$$

can be obtained from the definition (5), taking relations (8) and (9) into account. The Fourier transforms $\bar{\gamma}^{\omega}_{\alpha\beta}(R, R' \mid m, k_z)$ can be expressed, taking into account expression (8), in terms of the Fourier transform $U^{\omega}(R, R' \mid m, k_z)$ of the potential U:

$$\bar{\gamma}^{\omega}_{\alpha\beta}(R, R' \mid m, k_z) = (2\pi)^{-1} \int_0^{2\pi} d\varphi e^{im(\varphi-\varphi')} \bar{P}_{\nu\beta}(\varphi - \varphi') \times \sum_{n=-\infty}^{\infty} e^{in(\varphi-\varphi')} l^{\omega}_{\alpha\nu}\left(\frac{\partial}{\partial R} \middle| n, k_z\right) U^{\omega}(R, R' \mid n, k_z) \tag{15}$$

where the Fourier transform of the operator $\bar{L}_{\alpha\beta}$ has the form

$$l^{\omega}_{\alpha\beta}\left(\frac{\partial}{\partial R} \middle| n, k_z\right) = \rho\delta_{\alpha\beta}\left[c_l^2\left(\frac{\partial^2}{\partial R^2} + \frac{1}{R}\frac{\partial}{\partial R} - \frac{n^2}{R^2}\right) + \omega^2\right] - \frac{4\pi\rho}{A} \begin{Vmatrix} \frac{\partial^2}{\partial R^2} & \frac{in}{R}\left(\frac{\partial}{\partial R} - \frac{1}{R}\right) & -ik_z\frac{\partial}{\partial R} \\ \frac{in}{R}\left(\frac{\partial}{\partial R} \quad \frac{1}{R}\right) & \frac{1}{R}\left(\frac{\partial}{\partial R} - \frac{n^2}{R}\right) & \frac{nk_z}{R} \\ -ik_z\frac{\partial}{\partial R} & \frac{nk_z}{R} & -k_z^2 \end{Vmatrix} \tag{16}$$

We can construct the Fourier transform of the potential $U^{\omega}(R, R' \mid m, k_z)$ in several obvious ways. The simplest method consists of changing, in the known Fourier expansion /8/

$$U^{\omega}(x, y \mid k_z) = -A \sum_{\lambda=l,t}^{(-)} \int_{-\infty}^{\infty} \frac{\exp[-i(k_x x + k_y y)]}{k_x^2 + k_y^2 + q_\lambda^2} dk_x dk_y \tag{17}$$

$$q_\lambda = (k_z^2 - \omega^2/c_\lambda^2)^{1/2}$$

to cylindrical coordinates $x = R\cos\varphi$, $y = R\sin\varphi$, and converting the expansion (17) into a Fourier series in φ. As a result, using the well-known integral representation and addition theorems for Bessel functions /9, 10/, we obtain

$$U^{\omega}(R, R', \varphi - \varphi' \mid k_z) = -2A\omega^{-2} \sum_{\lambda=l,t} \sum_{n=-\infty}^{\infty} U_n^{(\lambda)\omega}(R, R' \mid k_z) e^{in(\varphi-\varphi')} \tag{18}$$

Here

$$U_n^{(\lambda)\omega}(R, R' \mid k_z) = I_n(q_\lambda R) K_n(q_\lambda R') \Theta(R' - R) + I_n(q_\lambda R') K_n(q_\lambda R) \Theta(R - R') \tag{19}$$

where $I_n(\xi)$ and $K_n(\xi)$ are modified Bessel functions of order n, of the first and second kind respectively. Substituting expression (19) into (15) and reducing the expression obtained, we finally have

$$\bar{\gamma}^{\omega}_{\alpha\beta}(R, R' \mid m, k_z) = \sum_{\lambda=l,t} \bar{\gamma}^{(\lambda)\omega}_{\alpha\beta}(R, R' \mid m, k_z) \tag{20}$$

where

$$\bar{\gamma}^{(\lambda)\omega}_{\alpha\beta}(R, R' \mid m, k_z) = [2\pi\rho\omega^2]^{-1}\left\{a^{(\lambda)}_{\alpha}(R \mid m, k_z) b^{(\lambda)}_{\beta}(R' \mid m, k_z)\theta(R - R') + b^{*(\lambda)}_{\alpha}(R \mid m, k_z) a^{*(\lambda)}_{\beta}(R' \mid m, k_z)\theta(R' - R) + \frac{\omega^2}{2c_\lambda^2}\frac{c_l^2 - c_\lambda^2}{c_l^2 - c_t^2} A^{(\lambda)}_{\alpha\beta}(R, R' \mid m, k_z)\right\} \tag{21}$$

The vectors $\mathbf{a}^{(\lambda)}$ and $\mathbf{b}^{(\lambda)}$ have the form

$$\mathbf{a}^{(\lambda)}(\xi \mid m, k_z) = \left\{-\frac{\partial}{\partial\xi}, -\frac{m}{\xi}, \frac{ik_z}{2}\right\} K_m(q_\lambda\xi)$$

$$\mathbf{b}^{(\lambda)}(\xi \mid m, k_z) = \left\{\frac{\partial}{\partial\xi}, -\frac{m}{\xi}, \frac{ik_z}{2}\right\} I_m(q_\lambda\xi)$$

and an asterisk denotes their complex conjugates. The diagonal matrix has the following components:

$$A_{RR}^{(\lambda)}(R, R' \mid m, k_z) = A_{\varphi\varphi}^{(\lambda)}(R, R' \mid m, k_z) =$$
$$[K_{m-1}(q_\lambda R) I_{m-1}(q_\lambda R') + K_{m+1}(q_\lambda R) I_{m+1}(q_\lambda R')] \theta(R - R') +$$
$$[I_{m-1}(q_\lambda R) K_{m-1}(q_\lambda R') + I_{m+1}(q_\lambda R) K_{m+1}(q_\lambda R')] \theta(R' - R)$$
$$A_{zz}^{(\lambda)}(R, R' \mid m, k_z) = 2[K_m(q_\lambda R) I_m(q_\lambda R') \theta(R - R') +$$
$$I_m(q_\lambda R) K_m(q_\lambda R') \theta(R' - R)]$$

It can be confirmed that the Fourier transforms (21) satisfy the reciprocity relations (7).

The Fourier transforms of the physical components of the displacement field defined by Eq.(1) can be written with the help of relation (20), and we then have

$$\bar{u}_\alpha(R, \varphi, z, t) = (2\pi)^{-2} \int_{-\infty}^{\infty} d\omega \int_{-\infty}^{\infty} dk_z \exp[i(\omega t - k_z z)] \times$$
$$\int dR' R' \sum_{m=-\infty}^{\infty} \bar{\gamma}_{\alpha\beta}^{\omega}(R, R' \mid m, k_z) \bar{f}_\beta{}^{\omega}(R' \mid m, k_z) \tag{22}$$

In the case when the fields are created in the medium by a system of dislocation loops, the tensor components of the force $\bar{f}_\beta{}^{\omega}(R, \varphi, z, t)$ are /11/:

$$f^\beta(\mathbf{y}, t) = \int_{-\infty}^{t} dt' \lambda^{\beta\gamma\nu\mu} \nabla_\gamma j_{\nu\mu}(\mathbf{y}, t') \tag{23}$$

where $j_{\nu\mu}(\mathbf{y}, t)$ is the dislocation flux density tensor /11, 12/. The spectral expansion of the physical components of the vector (23), taking Eq.(2) into account, has the form

$$\bar{f}_R{}^{\omega}(R \mid m, k_z) = \frac{2\rho c_t^2}{i\omega} \left\{ \left(\frac{c_l^2}{2c_t^2} - 1 \right) \frac{\partial}{\partial R} \bar{j}_{\nu\nu}^{\omega} + \right.$$
$$\left. \left(\frac{\partial}{\partial R} + \frac{1}{R} \right) \bar{j}_{RR}^{s\omega} + \frac{im}{R} \bar{j}_{R\varphi}^{s\omega} - ik_z \bar{j}_{Rz}^{s\omega} \right\}$$
$$\bar{f}_\varphi{}^{\omega}(R \mid m, k_z) = \frac{2\rho c_t^2}{i\omega} \left\{ \left(\frac{c_l^2}{2c_t^2} - 1 \right) \frac{im}{R} \bar{j}_{\nu\nu}^{\omega} + \right.$$
$$\left. \left(\frac{\partial}{\partial R} + \frac{2}{R} \right) \bar{j}_{R\varphi}^{s\omega} + \frac{im}{R} \bar{j}_{\varphi\varphi}^{s\omega} - ik_z \bar{j}_{\varphi z}^{s\omega} \right\}$$
$$\bar{f}_z{}^{\omega}(R \mid m, k_z) = \frac{2\rho c_t^2}{i\omega} \left\{ -ik_z \left(\frac{c_l^2}{2c_t^2} - 1 \right) \bar{j}_{\nu\nu}^{\omega} + \frac{\partial}{\partial R} \bar{j}_{Rz}^{s\omega} + \frac{im}{R} \bar{j}_{\varphi z}^{s\omega} - ik_z \bar{j}_{zz}^{s\omega} \right\}$$

Here $\bar{j}_{\nu\nu} = \bar{j}_{RR} + \bar{j}_{\varphi\varphi} + \bar{j}_{zz}$ is the trace of the tensor $\bar{j}_{\alpha\beta}(R, \varphi, z, t)$ and $\bar{j}_{\alpha\beta}^{s\omega}(R \mid m, k_z) = \frac{1}{2}(\bar{j}_{\alpha\beta}^{\omega} + \bar{j}_{\beta\alpha}^{\omega})$ are Fourier transforms of the physical components of the symmetric part of this tensor. For the filed of displacement rates of the points of the medium $\bar{v}_\alpha = \frac{\partial}{\partial t} \bar{u}_\alpha$ the transforms are: $\bar{v}_\alpha{}^{\omega}(R \mid m, k_z) = i\omega \bar{u}_\alpha{}^{\omega}(R \mid m, k_z)$. Fourier transforms of the stress field are obtained from Hooke's law for a medium with dislocations /11, 12/

$$\bar{\sigma}_{\alpha\beta}^{\omega}(R \mid m, k_z) = \bar{\lambda}_{\alpha\beta\gamma\delta} \bar{\nabla}_\gamma \bar{u}_\delta(R \mid m, k_z) + \frac{\rho}{i\omega} \{(c_l^2 - 2c_t^2) \delta_{\alpha\beta} \bar{j}_{\nu\nu}^{\omega} + 2c_t^2 \bar{j}_{\alpha\beta}^{s\omega}\}$$

When $\bar{j}_{\alpha\beta} = 0$, the above relation yields the well-known relation between the stresses and strains for a defect-free medium. These are given, for example, in the coordinate representation for the case of cylindrical coordinates in /1, 11/. The change to Fourier transforms in these expressions is made by formal replacement of the differentiation operators $\partial/\partial\varphi \to im$, $\partial/\partial z \to ik_z$. The obvious formulas for the transforms of the stress field of the type (22) are very bulky and are therefore not given here.

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A LINEAR THEORY OF DOUBLE-LAYER RESIN-METAL SHELLS*

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An asymptotic method is used to derive two-dimensional equations of double-layer shells of arbitrary form. The problem is split into two, simpler problems. A solution for a weak layer of slightly compressible elastic material, such as an elastomer, is obtained in general form and the solution for a two-layer shell reduces, as a result, to solving the problem of a stiff layer under a load which depends on the stress-strain state (SDS) of the weak layer. It is shown that in the case of a weak layer the laws of variation of the quantities required across the thickness may deviate significantly, depending on the dynamic properties of the load, from the laws accepted in the classical theory of shells.

The papers dealing with the problem in question concern themselves, as a rule, with the analysis of the equations of state /1, 2/, or with the study of SDS under kinematic-type conditions on the face surfaces of the shell, making certain assumptions /3/.

1. We shall assume, to fix our ideas, that the outer layer of the shell, of thickness $2h_1$, is composed of an incompressible elastic elastomer (we shall call it the soft layer), and an inner, metal layer of thickness $2h_2$ (we shall call it the stiff layer). The face surfaces of the two-layer shell are subjected to an arbitrary, static or dynamic load.

We will write the initial conditions for the elastomer layer in three-orthogonal coordinates $\alpha_1, \alpha_2, \alpha_3$ where α_1, α_2 are the lines of curvature of the middle surface of the layer and α_3 is a line orthogonal to them

$$\sigma_i^{(1)} = 2\mu e_i^{(1)} + p, \quad \sigma_{ij}^{(1)} = \mu\,(m_i^{(1)} + m_i^{(1)}) \tag{1.1}$$
$$\sigma_3^{(1)} = 2\mu\frac{\partial v_3^{(1)}}{\partial\alpha_3^{(1)}} + p, \qquad \sigma_{i3}^{(1)} = \mu\left(\frac{\partial v_i^{(1)}}{\partial\alpha_3^{(1)}} + g_i^{(1)}\right)$$
$$e_1^{(1)} + e_2^{(1)} + \frac{\partial v_3^{(1)}}{\partial\alpha_3^{(1)}} = 0 \tag{1.2}$$
$$L_i^{(1)} + \frac{\partial}{\partial\alpha_3^{(1)}}\sigma_{i3}^{(1)} + \rho_1\omega^2 v_i^{(1)} = 0 \quad -L^{(1)} + F^{(1)} + \frac{\partial\sigma_3^{(1)}}{\partial\alpha_3^{(1)}} + \rho_1\omega^2 v_3^{(1)} = 0 \tag{1.3}$$

Here (1.1) is the equation of state, (1.2) is the condition of incompressibility, (1.3) are the equations of motion, and

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